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A PERTURBATION TECHNIQUE FOR THE FINITE ELEMENT MODELLING OF NONDESTRUCTIVE EDDY CURRENT TESTING

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Abstract **– This paper deals with a new 3D finite element scheme for nondestructive eddy-current testing (ECT) problems. It concerns a perturbation technique applied to the magnetodynamic $h - \phi$ formulation. The unperturbed field (in the absence of the flaw) is computed conventionally in the complete domain. The source of the perturbation problem is determined by the projection of the unperturbed field in a relatively small region around the defect that depends on the work frequency. The discretisation of this reduced domain is adapted to the size of the defect and independent of the discretisation of the complete domain. The impedance change is computed by integrating only over the defect and a layer of elements in the reduced domain that touch its boundary. As test case, we consider the second eddy current benchmark problem proposed by the WFNDEC. It involves a pancake coil scanning the inner surface of a metal tube for the detection of flaws on the outer surface.**

Introduction

The nondestructive eddy-current testing (ECT) problems have been extensively studied in recent years. The ultimate goal is to determine the position and size of defects in conducting materials (inverse problem). However, a fast and accurate calculation of the probe response (forward problem) is often required for identifying the flaws from measured data. When the excitation is time-harmonic, the observed quantity is usually the impedance variation due to the presence of the defect.

Several variations of the volume integral method (VIM) have been reported in literature [1, 2]. Herein, defects can be represented by a distribution of current dipoles in its volume. A boundary element method, using VIM for describing the defect, is proposed in [3]. Only the crack is discretised. The calculations related to different probe positions are very fast. These techniques become extremely computationally expensive in case of more complicated geometries other than infinite stabs or tubes with homogeneous and linear material parameters. Another disadvantage of the method is the singularity of the Green's kernel.

The finite element method [4] allows to overcome these drawbacks. However, it may require a dense discretisation in the vicinity of the defect resulting in a large 3D mesh for the complete problem. The impedance change due to the defect is calculated as the difference of the impedance values with and without flaw. Furthermore, calculations for different probe positions are performed independently, which is time consuming.

A 3D $\boldsymbol{a}$-formulation based finite element scheme that calculates directly the distortion of the eddy-current due to a flaw is presented in [5]. Herein, the computation is split into a computation without flaw and a computation of the field distortion due to its presence. The unperturbed field (in the absence of the flaw) is calculated in a large region taking advantage of any symmetry or analytical solution, and applied as a source in the flaw for the second computation. The perturbed field can thus be determined in a reduced domain around the defect, what allows for a discretisation better adapted to the defect size, usually much smaller than the rest of the considered problem. The impedance change is computed by integrating only over the flaw region.

The present paper concerns the extension of this perturbation technique to the magnetodynamic $\boldsymbol{h}-\phi$ formulation. The coupling between the scalar potential ϕ in the defect and the magnetic field $\boldsymbol{h}$ in the conductor is described in detail. Further, an expression for the impedance change is derived.

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As test case, the second eddy current benchmark problem proposed by the WFNDEC [6] is considered. A study of the accuracy as a function of the extension of this reduced domain is performed.

Perturbation method

We consider a magnetodynamic problem in a bounded domain Ω (boundary Γ) of $\mathbb{R}^3$. The governing differential equations and constitutive laws of the magnetodynamic problem in Ω are

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{j}, \quad \operatorname{curl} \boldsymbol{e} = -\partial_t \boldsymbol{b}, \quad \operatorname{div} \boldsymbol{b} = 0\,, \quad \boldsymbol{b} = \mu \boldsymbol{h}\,, \quad \boldsymbol{j} = \sigma \boldsymbol{e}\,, \tag{1 a–e}$$

with $\boldsymbol{h}$ the magnetic field, $\boldsymbol{b}$ the magnetic flux density, $\boldsymbol{e}$ the electric field, $\boldsymbol{j}$ the current density, μ the magnetic permeability and σ the electric conductivity.

The eddy current conducting part of Ω is denoted Ω_c and the non-conducting one Ω_c^C ($\Omega = \Omega_c \cup \Omega_c^C$). Source conductors, with a given current density $\boldsymbol{j}_s$, are comprised in $\Omega_s \subset \Omega_c^C$. A flaw Ω_f with boundary Γ_f appears in Ω_c.

Let us suppose that the flaw has different conductivity σ_f and permeability μ_f than the host material. It is assumed that they are linear and isotropic. Particularising (1 a) and (1 b) for the unflawed and flawed arrangements, we obtain

$$\operatorname{curl} \boldsymbol{h}_u = \sigma \boldsymbol{e}_u\,, \qquad \operatorname{curl} \boldsymbol{e}_u = -\mu \partial_t \boldsymbol{h}_u\,, \tag{2 a–b}$$

$$\operatorname{curl} \boldsymbol{h}_f = \sigma_f \boldsymbol{e}_f\,, \qquad \operatorname{curl} \boldsymbol{e}_f = -\mu_f \partial_t \boldsymbol{h}_f\,, \tag{2 c–d}$$

where the subscripts u and f refer to the unflawed and flawed quantities, respectively. The source terms of the perturbed formulation are determined from the eddy-current distribution without defect.

Subtracting (2 a–b) from (2 c–d), respectively, it is easy to prove that

$$\operatorname{curl} \boldsymbol{h} = \sigma_f \boldsymbol{e} + \boldsymbol{j}_{sf}\,, \qquad \operatorname{curl} \boldsymbol{e} = -\mu_f \partial_t \boldsymbol{h} - \boldsymbol{k}_{sf}\,, \tag{3 a–b}$$

where $\boldsymbol{h} = \boldsymbol{h}_f - \boldsymbol{h}_u$ and $\boldsymbol{e} = \boldsymbol{e}_f - \boldsymbol{e}_u$ are the perturbations [5] and

$$\boldsymbol{j}_{sf} = (\sigma_f - \sigma)\boldsymbol{e}_u\,, \qquad \boldsymbol{k}_{sf} = (\mu_f - \mu)\partial_t \boldsymbol{h}_u\,, \tag{4 a–b}$$

are the incident electric and magnetic current densities that generate the perturbation of the field due to the change of σ and μ, respectively.

$\boldsymbol{h} - \phi$ magnetodynamic formulation

Adopting the magnetic field formulation, the general expression of the magnetic field $\boldsymbol{h}$ in Ω is $\boldsymbol{h} = \boldsymbol{h}_s + \boldsymbol{h}_r$, with $\boldsymbol{h}_s$ a source field in Ω satisfying $\operatorname{curl} \boldsymbol{h}_s = \boldsymbol{j}_s$ and $\boldsymbol{h}_r$ the reaction field in Ω_c. In the non-conducting regions Ω_c^C, the reaction field $\boldsymbol{h}_r$ can be derived from a scalar potential ϕ such as $\boldsymbol{h}_r = -\operatorname{grad} \phi$.

The $\boldsymbol{h} - \phi$ magnetodynamic formulation is obtained from the weak form of the Faraday law (1 a):

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h}')_\Omega + (\sigma^{-1} \operatorname{curl} \boldsymbol{h}, \operatorname{curl} \boldsymbol{h}')_{\Omega_c} + \langle \boldsymbol{n} \times \boldsymbol{e}, \boldsymbol{h}' \rangle_\Gamma = 0\,, \quad \forall \boldsymbol{h}' \in F_{h\phi}(\Omega) \tag{5}$$

where $\boldsymbol{n}$ is the outward unit normal vector on Γ, part of the boundary of Ω; $(\cdot,\cdot)_\Omega$ and $\langle \cdot,\cdot \rangle_\Gamma$ denote a volume integral in Ω and a surface integral on Γ of the product of their arguments; $F_{h\phi}(\Omega)$ is the function space defined on Ω and containing the basis functions for $\boldsymbol{h}$ (coupled to ϕ) as well as for the test function $\boldsymbol{h}'$ [7]. The trace of $\boldsymbol{e}$ is a constraint associated with Γ (this constraint can e.g. be associated with a homogeneous Neumann boundary condition or with a global quantity) [7, 8].

For the sake of simplicity, a zero-conductivity defect Ω_f with same permeability as Ω_c is assumed hereafter for the perturbation problem. The extension of the formulation to other cases is straightforward.

The unperturbed field $\boldsymbol{h}_u$ (with $\Omega_f \subset \Omega_c$) is obtained by particularising ($\boldsymbol{h} = \boldsymbol{h}_u$) and solving (5). This field $\boldsymbol{h}_u$ is then projected to a reduced domain $\Omega' \subset \Omega$ around the defect. Note that projecting only

$\boldsymbol{h}_u$ is not enough. This way the local current $\boldsymbol{j}_u = \mathrm{curl}\,\boldsymbol{h}_u$ will not be conserved. According to (4 a) with $\sigma_f = 0$, the perturbation current source $\boldsymbol{j}_{sf}$ is given by

$$\boldsymbol{j}_{sf} = \mathrm{curl}\,\boldsymbol{h}_{sf} = -\sigma \boldsymbol{e}_u = -\mathrm{curl}\,\boldsymbol{h}_u \quad \text{in } \Omega_f. \tag{6}$$

Furthermore, the trace of source field $\boldsymbol{h}_{sf} = -\boldsymbol{h}_u$, $\boldsymbol{n} \times \boldsymbol{h}_{sf}$, on Γ_f contributes to the exterior domain $\Omega' \backslash \Omega_f$. Indeed, the following interface condition has to be satisfied on Γ_f:

$$\boldsymbol{n} \cdot \boldsymbol{j} \mid_{\Gamma_f} = -\boldsymbol{n} \cdot \boldsymbol{j}_{sf} \mid_{\Gamma_f} , \tag{7}$$

which is equivalent to consider

$$\boldsymbol{n} \times \boldsymbol{h} \mid_{\Gamma_f} = -\boldsymbol{n} \times \mathrm{grad}\,\phi \mid_{\Gamma_f} + \boldsymbol{n} \times \boldsymbol{h}_{sf} \mid_{\Gamma_f} \ . \tag{8}$$

The source of the perturbation problem in Ω_f is calculated through a projection method in the reduced domain Ω' as

$$(\mathrm{curl}\,\boldsymbol{h}_{sf}, \mathrm{curl}\,\boldsymbol{h}')_{\Omega'} + (\mathrm{curl}\,\boldsymbol{h}_u, \mathrm{curl}\,\boldsymbol{h}')_{\Omega'} = 0\,, \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega'), \tag{9}$$

where a gauge condition using a tree-cotree method at the discrete level in Ω' is applied to ensure the uniqueness of the solution. The circulation of $\boldsymbol{h}_{sf}$ on the edges of $\Omega' \backslash \Omega_f$ is fixed to zero. For the sake of conciseness, hereafter we refer to Ω' as Ω.

The perturbation problem is completely characterised by (5) applied to the perturbation field $\boldsymbol{h}$ and taking into account (6) as follows:

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h}')_\Omega + (\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_c \backslash \Omega_f} + \partial_t(\mu \boldsymbol{h}_{sf}, \boldsymbol{h}')_\Omega + (\boldsymbol{j}_{sf}, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_f} = 0\,, \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega)\,. \tag{10}$$

Calculation of the impedance variation

The impedance of the exciting coil changes and this change allows to detect and characterise the defect. However, the change of the observed quantity is usually under 1% of the total value or even smaller in practical cases. The accurate calculation of this impedance variation ΔZ is thus crucial. Hereafter, it will be proved that ΔZ can be calculated by integrating the product of local quantities only over the flaw Ω_f and a layer of elements in $\Omega \backslash \Omega_f$ that touch the boundary Γ_f.

A suitable treatment of the surface integral term in (5) consists in naturally defining a global voltage V in a weak sense. We can define a global test function for $\boldsymbol{h}$ with a unit circulation along any current tube of the inductor so that the surface integral in (5) can be expressed as the product of a global voltage V and a unit global current $I(\mathrm{curl}\,\boldsymbol{h}')$[8].

Let us specify (5) for the unflawed problem, it holds

$$\partial_t(\mu \boldsymbol{h}_u, \boldsymbol{h}')_\Omega + (\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_u, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_c} = V_u\, I(\mathrm{curl}\,\boldsymbol{h}'), \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega). \tag{11}$$

Analogously, for the flawed problem, we can write

$$\partial_t(\mu \boldsymbol{h}_f, \boldsymbol{h}')_\Omega + (\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_c \backslash \Omega_f} + (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_f} = V_f\, I(\mathrm{curl}\,\boldsymbol{h}'), \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega), \tag{12}$$

where we have added the term $(\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_f}$ which is not cancelled as in the general case due to the imposed perturbation current in the flaw, i.e. $\mathrm{curl}\,\boldsymbol{h}' \neq 0$ in $\Omega_f \subset \Omega_c^C$.

Choosing as test functions $\boldsymbol{h}' = \boldsymbol{h}_f$ in (11) and $\boldsymbol{h}' = \boldsymbol{h}_u$ in (12) and subtracting (11) from (12), we obtain

$$\Delta V\, I = \Delta Z\, I^2 = -(\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_u, \mathrm{curl}\,\boldsymbol{h}_f)_{\Omega_f} + (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_u)_{\Omega_f} = (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_u)_{\Omega_f}\,, \tag{13}$$

where the first volume integral cancels because $\boldsymbol{h}_f$ is curl-free in Ω_f and I is the real current injected in the inductor.

The perturbed electric field $\boldsymbol{e}_f$ is not known in the flaw but can be calculated by means of (12) with $\boldsymbol{h}' = \boldsymbol{h}_{sf}$. This way $I(\mathrm{curl}\,\boldsymbol{h}') = 0$ and the impedance variation ΔZ is obtained as

$$\Delta Z\, I^2 = -(\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_{sf})_{\Omega_f} = \partial_t(\mu(\boldsymbol{h} + \boldsymbol{h}_{sf}), \boldsymbol{h}_{sf})_\Omega + (\sigma^{-1}\mathrm{curl}\,(\boldsymbol{h} + \boldsymbol{h}_{sf}), \mathrm{curl}\,\boldsymbol{h}_{sf})_{\Omega_c \backslash \Omega_f}\,, \tag{14}$$

where the domain of integration, at the discrete level, is actually limited to Ω_f and a layer of elements in $\Omega \backslash \Omega_f$ touching Γ_f due to the definition of $\boldsymbol{h}_{sf}$.

Application example

As numerical example, we consider the second eddy current benchmark problem proposed by the WFN-DEC [6]. It concerns an Inconel tube ($\sigma = 10^6$ S/m, inner diameter $D_i = 19.69$ mm, outer diameter $D_o = 22.23$ mm) with a defect on the outer surface and a pancake coil that scans the inner surface (Fig. 1). The flaw is $t = 1$ mm long in the axial direction, $w = 3$ mm wide in the radial direction and its depth h varies between 20% and 60% of the tube wall thickness. The coil (400 turns, inner diameter = 1 mm, outer diameter = 3 mm, height = 0.8 mm) carries an imposed sinusoidal current of 100 mA and frequency $f = 150$ kHz. The lift-off between the center of the inferior plan of the coil and the inner surface of the tube is 0.8 mm.

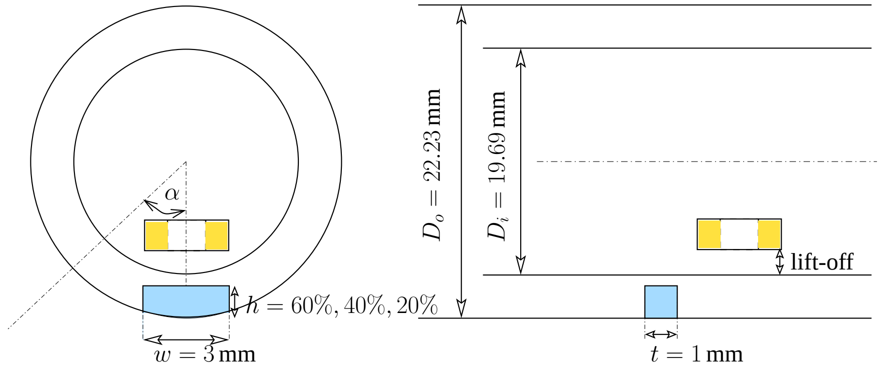


Fig. 1. Transversal (left) and longitudinal (right) cross sections of the Incodel tube with a defect on the outer surface and a pancake coil inside. The angle of rotation of the coil α is also shown.

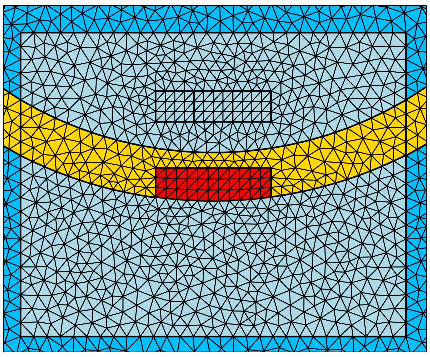


Fig. 2. Reduced domain Ω' of size 2.7 δ around the flaw used in the perturbation method. The source field is defined in the flaw, the excitation coil is taken as air.

First, we study the accuracy of the proposed method as a function of the size of the reduced domain Ω'. To this purpose, we consider a 2D model of the problem and we vary the dimensions of Ω' around the defect in terms of multiples of the skin depth $\delta = 1/\sqrt{\pi f \mu \sigma} = 1.3$ mm of the tube. We calculate the impedance variation ΔZ both in the conventional way (solving the unflawed and the flawed problem consecutively and performing the difference between two impedance values) and with the proposed perturbation method (integrating directly in a sub-domain of Ω').

In order to avoid numerical errors due to the discretisation, the conventional technique requires exactly the same mesh for the unflawed and flawed problem. Nevertheless the proposed perturbation technique can be applied using too independent meshes: a mesh of the whole domain without considering the defect and a mesh of the reduced domain without the explicit presence of the excitation coil. For the sake of a fair comparison, the error is calculated using exactly the same mesh for the two methods

(Fig. 2). In the perturbation method, the source field is given by an equivalent current source in the flaw itself.

The impedance of the coil obtained for the unperturbed problem is $25.5 + \imath\, 210.75\,\Omega$. The relative error (%) in the real and imaginary parts of ΔZ is depicted in Figs. 3a and 3b, respectively. We can observe that in both cases the relative error is smaller than 1% when the distance between the border of the reduced domain and the crack is larger than $2.7\,\delta$.

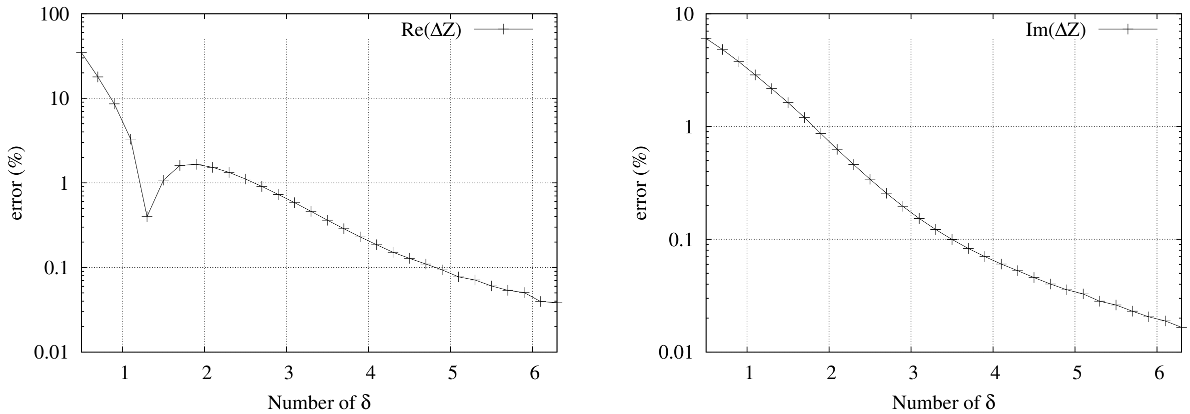


Fig. 3. Relative error (%) in the real and imaginary parts of ΔZ as a function of the dimensions of Ω', depending on a multiple of the skin depth δ.

In order to validate the numerical model, our 3D results are compared with those presented in [9]. The locus and magnitude of the impedance change for an axial scan and different depths of the flaw are represented in Figs. 4a and 4b for a rotation angle $\alpha = 0°$ and in Figs. 5a and 5b for a rotation angle $\alpha = 20°$. An excellent agreement between our results and those presented in [9] is observed.

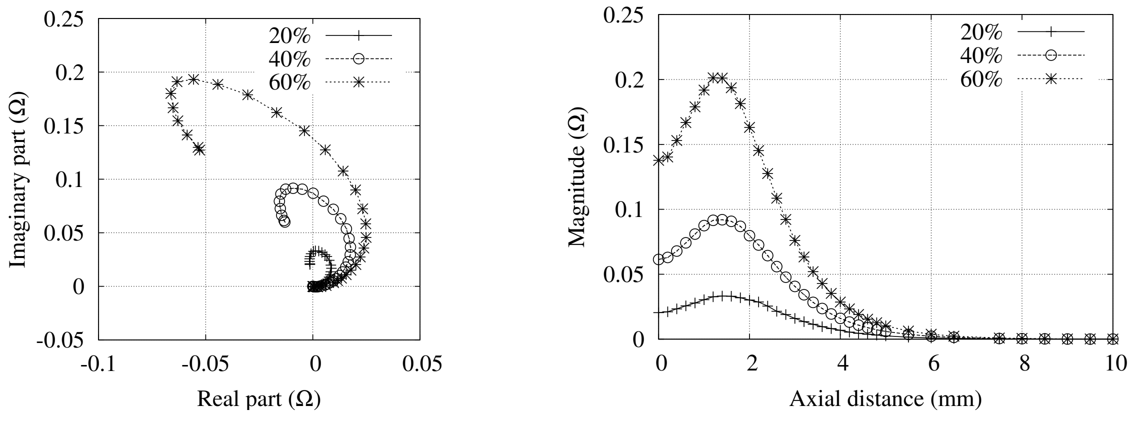
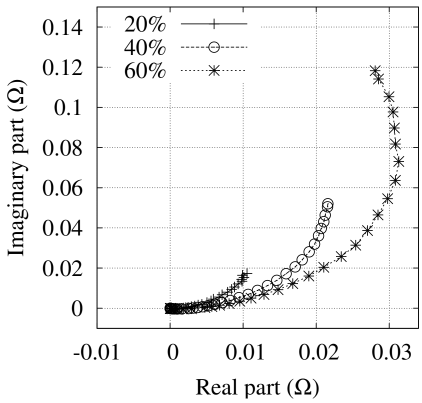


Fig. 4. Locus (left) and magnitude (right) of the impedance change ΔZ for an axial scan with $\alpha = 0°$.

Conclusions

A 3D FE perturbation technique based on the $\boldsymbol{h} - \phi$ magnetodynamic formulation has been elaborated. The unperturbed field is calculated conventionally in the complete domain taking advantage of any symmetry or analytical solution (if available) and applied as a source in the flaw. Next the perturbed field is



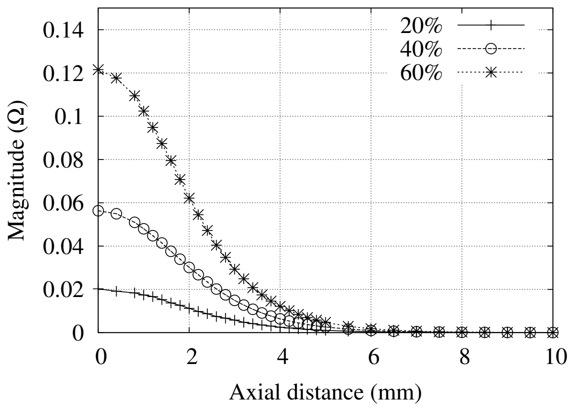


Fig. 5. Locus (left) and magnitude (right) of the impedance change ΔZ for an axial scan with $\alpha = 20^\circ$.

determined in a reduced domain surrounding the defect. The discretisation is thus chosen independently of the dimensions of the excitation coil and the specimen under study. Furthermore, the impedance variation due to the presence of the flaw is calculated by performing an integral over the defect and a layer of elements in the exterior domain that touch its boundary.

The accuracy of the model has been evidenced by comparing the results obtained for different dimensions of the reduced domain to those achieved in the conventional way. The field distortion can actually be neglected beyond a distance $2.7\,\delta$ around the defect. Finally, the solution of the 3D eddy current benchmark problem validates the presented perturbation scheme.

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